# CALCULATION OF TEMPERATURE DISTRIBUTION FOR VISCOUS INCOMPRESSIBLE FLOW WITH CONSTANT THERMAL CONDUCTIVITY

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**Dedicated To**

***My Beloved Family***

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# Nomenclature

| | |
|---|---|
| ST: | Stretching sheet |
| AMT: | Across mass transfer |
| MHD: | Magneto hydro dynamics |
| SC: | Slip condition |
| N.S. SC: | Navier Stokes Slip condition |
| VIF: | Viscous incompressible fluid |
| MF: | Magnetic field |
| SM: | Shooting Method |
| SS: | Shear Stress |
| Cvg: | Convergence |
| HTP: | Heat transfer phenomenon |

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# Abstract

The investigation of heat and mass transfer is the top consideration for mathematicians. In light of its vital applications in the field of polymer engineering, the consideration of heat transaction procedure from a stretching sheet to the boundary layer fluid has been developed completely. The aim of work is to attain the temperature profile by varying the different parameters having constant thermal conductivity .Generally , 3D flow of a viscous incompressible fluid has been assessed in the existence of AMT phenomenon. The impact of injection and suction has been deliberated in detail. The fundamental system has the equations, i.e. Momentum equation (M.eq) and Energy equation (E.eq) which are transformed into Ordinary Differential equations (ODEs) by means of similarity alteration. The solution of the problem has been achieved by utilizing analytical method i.e. homotopy analysis method (HAM). The precision of this technique is occurred in literature for further problems. The graphs of various parameters present the solution of the problem.

# Chapter # 1

# Basic Concepts And Literature Review

## 1.1 Introduction

Fluid dynamics is the part of mathematics which causes us to think about the fluid movement. It incorporates the movement of gases just as the movement of fluids. Because of the vast applications in the field and industry, researchers have performed unlimited research on it. A vital research is completed for the BLF around ST . From last few years, the study of heat has been source of excessive attention for mathematicians. Calculation of viscous drag on ST is the main part of these problems. Day by day researchers are working to build new techniques to decrease and overcome the viscous hindrance on rigid boundary. However fluid mechanics has become the most stimulating sub area of mathematics. Wear on, a number of theories and researches have been done, which are good accumulation. Because of its vital applications in the field of polymer engineering, the investigation of heat exchange procedure from a ST to the encompassing liquid has developed significantly. Various composing has been dedicated to the examination of most appropriate flow which can generate a most extreme proportion of heat from the hot boundary to the encompassing fluid. Analysis of heat interchange of a viscous fluid flow around a ST was first concentrated by Crane [1].He considered a 2D BLF of a viscous liquid over a ST and displayed a closed structure arrangement. In any case, the composition needs in giving a satisfactory material on 3D flows with heat conversation phenomenon, yet at the same time, a lot of work has been presented. For example, the 3D heat flow in a duct of subsidiary extending wall in a pivoting medium.

In genuine practice, it is seen that such sort of studies are progressively convenient in limited area. In this concern Munawar [2] researched the impact of clasping on the hydro dynamic stream in a turning duct of subordinate extending wall. As of late, Mehmood [3] examined the 3D stream in a permeable duct of subordinate distending wall .The author presented the AMT phenomenon by taking infusion at upper part and suction at lower part of the plate and effectively displayed that by compelling appropriate number of terms of injection and suction ratio at the higher and subsidiary part of the plates, the viscous drag on ST can definitely be précised and accustomed. HAM is used for solving the fundamental nonlinear diff .eqs. HAM is extensively used by the researchers [4],[5],[6],[7],[8],[9],[10]. Additionally, Mehmood [11]examined the heat move exploration in a comprehensive 3D stream of a viscous fluid in a duct expecting the top side of the duct as a permeable sheet exposed to constant infusion. Accordingly, it is required to implement some appropriate stream suppositions under which the impact of viscous dissipation can be decreased. Some increasingly vital commitments are referencing here. Ziabakhsh [12] examined the analytic solution for chemically reactive species over nonlinear ST. Tamayol [13] deliberated the thermal analysis in a permeable medium. Also Moghim [14] worked on the application of HAM. Seth [15] imparted the exploration of MHD flow nearby a nonlinear ST by means of N.S SC. Khan [16] conferred the effects of viscidness in a permeable medium. Cortell [17] studied the arithmetical solution of an un-steady 2D nano flow over a ST. Furthermore [18], [19], [20], [21],[22], [23], [24] [25], [26], [27],[28] and [29] have great worth in research field.

Chapter 1 conveys the fundamental ideas and definitions that are utilized in next sections. Essential administering conditions depicting the stream of the viscous fluid are planned by utilizing central laws of preservation of mass and energy. Some basic amounts of physical intrigue, for example skin friction coefficient, Maxwell conditions and dimensionless numbers are additionally presented in this part.

Chapter 2 manages the explanatory investigation of viscous flow inside a duct restricted by two sheets. The upper part of it is exposed to the consistent infusion and uniform suction is taken at the subordinate ST. This phenomenon is known as AMT. The HAM is utilized to get an investigative answer for two or three nonlinear diff. eqs. The arrangement is examined in detail by plotting diagrams and tables. The consideration exhibited in this part is a survey of an article by [30].

Chapter 3 is the extension of the past issue by taking the SC on the bottom side of the duct within the sight of magnetic flied impact. It is well known truth that the skin friction increases in the hydro magnetic flows. Therefore the slip condition is familiarized to diminish the wall shear force on the ST and calculate the temperature distribution for viscous incompressible flow with constant thermal conductivity. The impact of various factors on temperature has been remarked. The foremost nonlinear diff. eqs are resolved with HAM to get an analytic solution.

## 1.2 Basic definitions

### 1.2.1 Flow

The relentlessly and consistently motion of a liquid or gas in a stream is called flow.

### 1.2.2 Fluid

A substance that has no fixed shape and yields effectively to external pressure.

### 1.2.3 Kinematic viscosity

It is denoted by $\nu$ and mathematically defined by

$$\nu = \frac{\mu}{\rho} \tag{1.1}$$

### 1.2.4 Incompressible fluid

For incompressible fluid $\rho$ is constant.

$$\nabla . \mathrm{V} = 0 \tag{1.2}$$

### 1.2.5 Unsteady and steady flows

The stream for which all the liquid properties rely upon time is called unsteady flow. In fact, more or less all flows are unsteady in some sense. Otherwise steady flow.

i.e $$\frac{\partial L}{\partial t} = 0 \tag{1.3}$$

Where L stands for any fluid property.

### 1.2.6 3D flow

3D flow denotes the number of space coordinates required to define a flow.

### 1.2.7 Turbulent and laminar flows

The flow in which the velocity of the fluid is constant at any point of the fluid is called laminar flow. In this flow each particle of the fluid flows in smoot path. While, the irregular flow is called turbulent flow and the velocity of fluid does not remain constant.

## 1.2.8 Law of conservation of mass

The mass of the structure must stay consistent after some time, as structure's mass can't change, so sum can't be included nor evacuated. Consequently, the degree of mass is restricted after some time**.** In fluid dynamics, law of conservation of mass expresses that the net difference in mass inside a given control volume is equivalent to the contrast between the amount of mass entering and that leaving the size controller.

It is characterized as

$$\frac{\partial}{\partial t}\left(\int_V \rho dV\right)+\int_S (n.\rho V)\, dS = 0 \tag{1.4}$$

in which the initial part signifies change rate of mass within the size controller and the later one implies net mass rate change out over the control surface. The assimilated element stated that the significant part of the flow i.e. dS, ρ stands for the density and n is the component normal to the plane.

### 1.2.9 Momentum law of conservation

This law states that for two colliding bodies in an isolated system, the all-out momentum when the impact is equivalent, unless an external force is exerted on it. . Arithmetically it is denoted as

$$\mathbf{F} = m\mathbf{a} \tag{1.5}$$

**F** is used for the applied force, **a** denotes the acceleration and m the mass of the fluid element . From eq. (1.5) we have

$$\rho\frac{dV}{dt} = f = f_{body} + f_{surface} \tag{1.6}$$

Here, f is the applied force which is additionally part into two sections; the body constrain and the surface power. First one are those which apply all in all mass of the liquid component and the later are those which apply weight on the sides of the liquid component.

### 1.2.10 Newton's law of viscosity

The pressure among nearby liquid layers is connection to the speed inclination between the two layers. Mathematically,

$$\tau \propto \frac{du}{dy} \tag{1.7}$$

And

$$\tau = \mu \frac{du}{dy} \tag{1.8}$$

Where, τ is the tangential stress, μ the constant of proportionality and du/dy the rate of deformation. The fluids which follow the viscosity law of Newton are called Newtonian fluid (NF) and that do not follow the viscosity law of Newton are called Non Newtonian fluids (NNF). Water ,oil, alcohol etc are NF while suspensions and gels are NNF.

## 1.2.11 Navier stokes equations

For incompressible viscous fluids, the vector form of N.S. eqs is

$$\rho \frac{\partial V}{\partial t} + \rho (V.\nabla) V = \nabla.T + \rho b \tag{1.9}$$

Here, density is denoted by $\rho$ , T stands for Cauchy tensor, b the body constrain and V shows velocity of the flow. The Cont.eq is

$$\nabla .V=0 \tag{1.10}$$

N.S eq of motion for 3D VIF is

$$\rho \frac{Du_i}{Dt} = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{1.11}$$

And the N.S eq for 3D VCF is

$$\rho \frac{Du_i}{Dt} = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right)\right] \tag{1.12}$$

### 1.2.12 Coefficient of skin friction

It is because of the fluid's friction close to the outside of an object that is traveling through it. The symbol used to denote it is $\tau_w$ and mathematically represented as

$$\tau_w = \lim_{y \to 0}\left(\mu \frac{du}{dy}\right) \tag{1.13}$$

And the coefficient of skin friction is defined as

$$C_f = \frac{\tau_w}{\rho U_\infty^{\ 2}} \tag{1.14}$$

### 1.2.13 No-slip condition

For viscous fluids it is assumed that at a strong bound, the liquid will have zero speed near to the bound.

### 1.2.14 Slip condition

The slip wall condition is for the instance when viscous effects at the wall are minor or the mesh size is greater than the boundary layer thickness. The boundary resists the slipping with a shear force relational to the slip velocity. Along these lines we can compose it as

$$u - U_w = \lambda \frac{\partial u}{\partial n} \tag{1.15}$$

Where, n is the coordinate normal to the wall, λ the slip length and $U_w$ the velocity of the wall.

### 1.2.15 Magnetohydrodynamics

MHD for short, is the part of fluid mechanics in which the fluid is electrically leading and travels in an attractive field. The essential hypothesis behind MHD is that attractive field actuates current in a moving conductive liquid, which hence makes controls on the liquid and besides changes the attractive field itself. The conditions that depict the movement of a leading fluid in a irresistible field are known as Maxwell's eqs.

### 1.2.16 Equations of Maxwell in MHD

These are a bundle of four incomplete distinctive conditions that depict how electric and MF proliferate, communicate and are influenced by substances.

### 1.2.17 Gauss's law for electricity

It expresses that in a closed region the over-all of the electrical fluidity coming of it is equivalent to voltage surrounded divided by the permittivity. Its numerical expression is

$$\nabla.\mathbf{E}=\frac{\rho}{\varepsilon_0} \tag{1.16}$$

Where, $\varepsilon_0$ the electric permittivity which is constant. **E** is the electric flux, and ρ the total voltage.

### 1.2.18 Gauss's law of magnetism

It is defined as in a closed region the total magnetic flux coming out is equal to zero. Arithmetically ,

$$\nabla.\mathbf{B} = 0 \tag{1.17}$$

Where, **B** is the total magnetic flux.

### 1.2.19 Faraday's law of induction

It is termed as in a closed path, the EMF made is correspondent to the time rate of progress of irresistible transition all through the way. Its mathematical formulation is

$$\nabla\times\mathbf{E}=-\frac{\partial B}{\partial t} \tag{1.18}$$

This expression is also known as Maxwell Faraday eq.

## 1.2.20 Ampere's law

The mathematical form of Ampere's law is given by

$$\nabla \times \mathbf{B} = \mu_0 \left( J + \varepsilon_0 \frac{\partial E}{\partial t} \right) \tag{1.19}$$

Here, electric field E is constant with respect to time then we get

$$\nabla \times \mathrm{B} = \mu_0 \mathrm{J} \tag{1.20}$$

Where **B** is the magnetic flux, **J** the current density and μ0 the permeability of magnetic flux.

## 1.2.21 Hartmann number

It is expressed as

$$M = B_0 L \sqrt{\frac{\sigma}{\mu}} \tag{1.21}$$

Where, L the characteristic length of the geometry, σ electrical conductivity and μ the viscosity of fluid. It is a dimensionless quantity.

## 1.2.22 Reynolds number

It is used to demonstrate whether fluid stream past a body or in a pipe is steady or turbulent**.**

It is also a dimensionless quantity and represented by Re

i.e

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{U L}{\nu} \tag{1.22}$$

where , L denotes the extent of the flow and U the free stream velocity of the flow. In other words , it is the quotient to the inertial force to viscous force. The flows in which Reynolds number is small are termed as laminar and the flows that occur at large Reynolds number are turbulent.

### 1.2.23 Knudsen number

It is expressed as the connection of the atomic mean free way length to a delegate physical length scale. It is denoted by the symbol Kn and it is a dimensionless quantity.

$$Kn = \frac{\lambda}{h} \tag{1.23}$$

Where, h is the characteristic physical length scale and. λ is the free path length.

### 1.2.24 Eckert number

It is relation of the heat degeneracy potential to the advective transport. Being a dimensionless quantity it is used in Continuum Mechanics and defined as

$$E_c = \frac{u^2}{\Delta T \times C_p} \tag{1.24}$$

Where,

- $\Delta T$ is contrast between divider temperature and neighborhood temperature.
- $u^2$ is the neighborhood stream speed of the continuum,
- $C_p$ is the constant-pressure local specific heat of the continuum,

### 1.2.25 Prandtl number

The term defined as fraction of the viscous dispersion ratio to the thermic dispersion relation. It is denoted by Pr and it is a dimensionless number given by

$$\Pr = \frac{\nu}{\alpha} = \frac{\mu / \rho}{k / C_p \rho} \tag{1.25}$$

Where $\alpha$ is thermal diffusivity, and $\nu$ is a kinematics viscosity.

## 1.2.26 Thermal conductivity

Thermal conductivity (TC) is a proportion of its capacity to bearing heat. It is ordinarily signified by $\mathrm{k}, \lambda$ or $\chi$ . The major eq for thermal conductivity is

$$q = -k\nabla T \qquad (1.26)$$

Where, $\nabla T$ is the heat grade, k is the TC and q is the heat flux

# Chapter # 2

# Across Mass Transfer Phenomenon in MHD Flow with Slips Conditions

## 2.1 Introduction

This part examines the AMT phenomenon impacts on a viscous fluid in which the lower plate is extending straightly. The process of AMT is presented by taking suction at the one sheet and infusion on the other all the while. The governing conditions for a 3D viscous incompressible stream are standardized with the assistance of similarity factors. An investigative arrangement is gotten for the nonlinear diff. eqs by utilizing HAM. The convergence of solution is analyzed quickly. HAM is effective for the entire range of the included factors. Precision of the solution will be contrast with numerical technique. The result is discussed through charts and tables to look at the impacts of different parameters on skin friction coefficient and speed profile . This work is a concise review of an ongoing paper by[30].

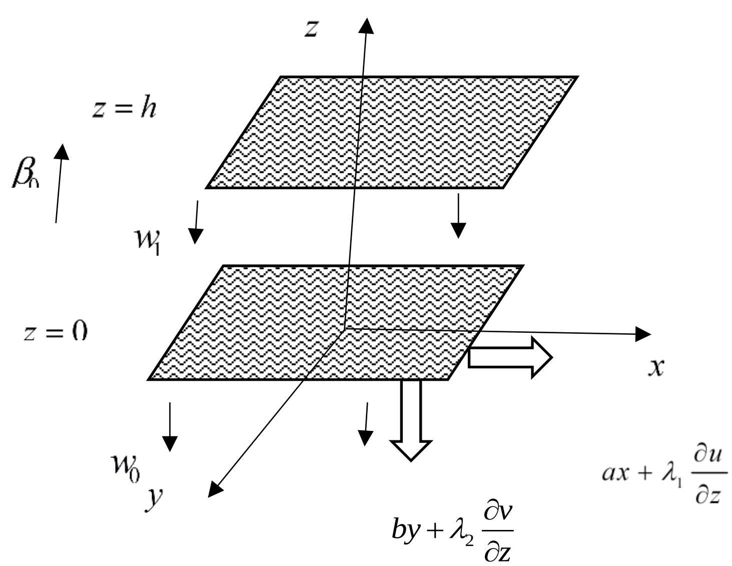


**Fig 2.1.** Diagram of the stream formation

## 2.2 Statement of the problem

Consider a steady VIF streaming among two parallel permeable plates at distance h. The upper plate is arranged at $z = h$, and the subordinate plate is situated at $z = 0$. The subordinate sheet is extending in two opposite ways at distinctive rates. For this situation the stream setup is 3D just as three-directional. At low Re the impact of prompted MF can be ignored. The administering conditions are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \vartheta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma}{\rho} {\beta_0}^2 u \tag{2.2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \vartheta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\sigma}{\rho} {\beta_0}^2 v \tag{2.3}$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \vartheta \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{2.4}$$

Applied to the boundary conditions given below

$$z = 0: \quad u = ax + \lambda_1 \frac{\partial u}{\partial z}, \mathrm{v} = \lambda_2 \frac{\partial v}{\partial z}, \quad w = -w_0 \tag{2.5}$$

$$z = h: \quad u = 0, \mathrm{v} = 0, \quad w = -w_1 \tag{2.6}$$

Here $u, v$ and $w$ are velocity constituents, $p$ the pressure, $\lambda_1$ is the slide extent alongside x-axis, $\lambda_2$ is the extent alongside y-axis, $h$ the distance between the walls of the channel, $a$ and $b$ are stretching proportions towards x and y axis respectively. The resulting dimensionless amounts are utilized for normalization:

$$\eta = \frac{z}{h}, u = ax F'(\eta), v = ay G'(\eta), w = -ah(F + G) \tag{2.7}$$

By using eq. (2.7) we obtained the coupled nonlinear diff eqs

$$F^{iv} - Re\,(M^2 F'' + F' F'' - F F''' - F'' G' - F''' G) = 0 \tag{2.8}$$

$$G^{iv} - Re\,(M^2 G'' + G' G'' - F G''' - F' G'' - G''' G) = 0 \tag{2.9}$$

Together with given bound

$$F'(0) = Kn_1 F''(0) + 1, \quad G'(0) = Kn_2 G''(0) + \beta, \quad G(0) + F(0) = w_0$$

$$F'(1) = 0\ ,\ G'(1) = 0\ ,\ G(1) + F(1) = w_1 \tag{2.10}$$

Where $\beta = b/a$ is the proportion of velocity grades , $\mathrm{Re} = ah^2/\upsilon$ symbolize the Re, $M = \beta_0\sqrt{\sigma/(a\rho)}$ is the Hartmann number. $w_0 = w_0^*/(ah)$ and $w_1 = w_1^*/(ah)$ are clout and inoculation factors and The skin friction constants are

$$C_{Fx} = \frac{T_{xz}}{\rho\,(ax^2)} = \frac{1}{Re_x}\ F''(0)\ ,\ C_{Gy} = \frac{T_{yz}}{\rho\,(ay^2)} = \frac{1}{Re_y}\ G''(0) \tag{2.11}$$

And the total shear force on the surface is

$$\tau = \sqrt{\left(\mathrm{Re}_y\, C_{Fy}\right)^2 + \left(\mathrm{Re}_x\, C_{Fx}\right)^2} = \sqrt{\left(\mathrm{G}''(0)\right)^2 + \left(\mathrm{F}''(0)\right)^2} \tag{2.12}$$

Here $\mathrm{Re}_x = axh/\upsilon$ and $\mathrm{Re}_y = ayh/\upsilon$ are local Reynold numbers.

## 2.3 Analytic solution by HAM

The eqs (2.8) and (2.9) with bounds given in (2.10) is solved by HAM. We pick the accompanying preliminary predicts

$$\begin{aligned} F_0(\eta) = {} & \frac{1}{2}\ w_0 + \left(1 + \frac{2\,Kn_1}{4\,Kn_1 + 1}\left(\frac{3}{2}\ w_1 - \frac{3}{2}\ w_0 - 2\right)\right)\eta \\ & + \left(\frac{1}{4\,Kn_1 + 1}\left(\frac{3}{2}\ w_1 - \frac{3}{2}\ w_0 - 2\right)\right)\eta^2 \\ & + \left(-\frac{1}{3} - \frac{2\,(Kn_1 + 1)}{4\,Kn_1 + 1}\left(\frac{1}{2}\ w_1 - \frac{1}{2}\ w_0 - \frac{2}{3}\right)\right)\eta^3 \end{aligned} \tag{2.13}$$

$$\begin{aligned} G_0(\eta) = {} & \frac{1}{2}\ w_0 + \left(\beta + \frac{2\,Kn_2}{4\,Kn_2 + 1}\left(\frac{3}{2}\ w_1 - \frac{3}{2}\ w_0 - 2\,\beta\right)\right)\eta \\ & + \left(\frac{1}{4\,Kn_2 + 1}\left(\frac{3}{2}\ w_1 - \frac{3}{2}\ w_0 - 2\,\beta\right)\right)\eta^2 \\ & + \left(-\frac{\beta}{3} - \frac{2\,(Kn_2 + 1)}{4\,Kn_2 + 1}\left(\frac{1}{2}\ w_1 - \frac{1}{2}\ w_0 - \frac{2}{3}\,\beta\right)\right)\eta^3 \end{aligned} \tag{2.14}$$

And for linear operator we take

$$L = \frac{\partial^4}{\partial \eta^4}$$

### 2.3.1 Mth-Order deformation of HAM

We write the mth-order distortion eqs with given bound at $m \geq 1$ as

$$L\,[\,F_m(\eta) - \chi_m F_{m-1}(\eta)\,] = \nabla_1 R_{1m}(\eta), \quad L\,[\,G_m(\eta) - \chi_m G_{m-1}(\eta)\,] = \nabla_2 R_{2m}(\eta), \tag{2.15}$$

$$F'_m(0) = Kn_1 F''_m(0), \quad G'_m(0) = Kn_2 G''_m(0), \quad G_m(0) + F_m(0) = 0 \tag{2.16}$$

$$G'_m(1) = F'_m(1) = G_m(1) + F_m(1) = 0,$$

Where

$$\begin{aligned} R_{1m}(\eta) = {} & F^{iv}{}_{m-1}(\eta) - Re\,(\,M^2 F''{}_{m-1}(\eta) \\ & + \sum_{k=0}^{m-1} F'{}_{m-1-k}(\eta)\, F''{}_k(\eta) - F_{m-1-k}(\eta)\, F''{}_k(\eta) - G_{m-1-k}(\eta)\; F'''{}_k(\eta) \\ & - G'{}_{m-1-k}(\eta)\, F''{}_k(\eta)\,) \\ R_{2m}(\eta) = {} & G^{iv}{}_{m-1}(\eta) - Re\,(\,M^2 G''{}_{m-1}(\eta) \\ & + \sum_{k=0}^{m-1} G'{}_{m-1-k}(\eta)\, G''{}_k(\eta) - F_{m-1-k}(\eta)\, G''{}_k(\eta) - G_{m-1-k}(\eta)\; G'''{}_k(\eta) \\ & - F'{}_{m-1-k}(\eta)\, G''{}_k(\eta)\,) \end{aligned} \tag{2.17}$$

$$\chi_m = \begin{cases} 0, k \leq 1, \\ 1, k > 1, \end{cases} \tag{2.18}$$

The solutions of these eqs written as of unbounded sequence are

$$F(\eta) = F_0(\eta) + \textstyle\sum_{m=1}^{\infty} F_m(\eta), \quad G(\eta) = G_0(\eta) + \textstyle\sum_{m=1}^{\infty} G_m(\eta), \tag{2.19}$$

Where $F_m(\eta)$ and $G_m(\eta)$ are the solutions of mth-order distortion eq (2.15) for $m \geq 1$.

### 2.3.2 Convergence of HAM

In the HAM, the cvg of the sequence solutions depends upon the cvg directing factors $h_1$ and $h_2$. The accuracy of result is obtained by figuring the arrangements at various commands of estimation. It is perceived that the amendments to the solutions become slight after the 5th estimation. This displays the cvg of our result.

The precision of HAM result of eqs (2.8) and (2.9) is assessed through contrasts with numerical solution attained by the SM (table 2.1).

**Table 2.1:** Effects of numerous constraints on the skin resistance constant $f''(0)$ when $\mathrm{Re} = 2$

| | | | $Kn_1 = 0$ | $Kn_1 = 1$ | $Kn_1 = 2$ |
|---|---|---|---|---|---|
| $M$ | $w_0$ | $w_1$ | HAM results | HAM results | HAM results |
| 0 | 0.5 | 0.5 | -4.76606 | -0.82106 | -0.45070 |
| 1.0 | 0.5 | 0.5 | -5.01409 | -0.82882 | -0.45306 |
| 3.0 | 0.5 | 0.5 | -6.68011 | -0.86734 | -0.46443 |
| 5.0 | 0.5 | 0.5 | -9.10245 | -0.89991 | -0.47364 |
| 0.5 | 0.5 | 0.5 | -2.74718 | -0.50387 | -0.27790 |
| 0.5 | 0 | 0.5 | -4.82916 | -0.82310 | -0.45132 |
| 0.5 | 0.5 | 0.5 | -7.35873 | -1.16144 | -0.63315 |
| 0.5 | 1.0 | 0.5 | -14.06255 | -1.89443 | -1.02146 |
| 0.5 | 2.0 | 0 | -6.41152 | -1.12348 | -0.61871 |
| 0.5 | 0.5 | 1.0 | -3.04058 | -0.50378 | -0.27508 |
| 0.5 | 0.5 | 2.0 | 1.16871 | 0.18374 | 0.09960 |

## 2.4 Outcomes and Discussion

Figure 2.1 represents the influence of $Kn_1$ and $Kn_2$ on the velocity, $f'$ in the existence of AMT phenomenon. It is seen that the speed of the liquid adjacent the ST decreases as the Kn increases .The reason is the decrease in the force transport because of expansion of slide length. At no-slip condition ($Kn_1 = Kn_2 = 0$), an opposite flow occurs owing to the skin resistance at the ST rises. But when the Knudsen number increases, this reverse flow disappears.

Fig 2.2 is drawing to observe the impact of AMT phenomenon under the SC. It is noted that fluid velocity increases as the injection velocity at the upper and suction velocity at the lower sheet rises. This expansion in the speed profile results in the advancement of the viscous hindrance on the ST . Moreover, it is noticed in fig 2.3 that the viscid hindrance upturns significantly as $w_0$ and $w_1$ increases with small values of the slip length, though, as the slip extent upturns, the augmentation in the hindrance control drops and wiped out after $Kn_1 = Kn_2 = 1.5$.

As in given table 2.1 , to minimalize the viscid resistance, we see that for a fixed estimation of $w_0$ we can determine the value of $w_1$. In order to get the result of expanding hydro attractive influence on the stream in the presence of SC fig 2.4 is given. The fig shows two variety drifts in the speed profile as the $M$ expands, the speed profile increments in the lower some portion of the channel ( $0 < \eta \leq 0.35$ ) and drop out in the upper part of the channel i.e.( $\eta > 0.35$ ). Fig 2.5 clarifies the SS being a function of Kn for various values of M . As $M$ upturns, the SS also rises which is a direct result of the expansion in the resistive power. As a result the total SS at the ST surges with growing Kn. This revenues that the Kn apply a substantial influence on the viscid slog in hydro magnetic flows in the occurrence of SC.

Fig 2.6 explains the effect of Re on the velocity contour. As $\mathrm{Re}$ rises, the velocity reveals two variant behavior. The velocity contour declines as $\mathrm{Re}$ upturns in the lower region of the duct while the contrary pattern is identified in the upper section of the channel. This habits of the speed profile is inferable from the developing inertial impact created by distending. Fig 2.7 establishes that the overall SS on the ST rises as the $\mathrm{Re}$ rises. Since the extent of the wall skin resistance increments because of the developing distinction in the speeds of the mass of the liquid. At that point with aggregate slip parameter, the expansion in the viscid delay the ST is moderate down.

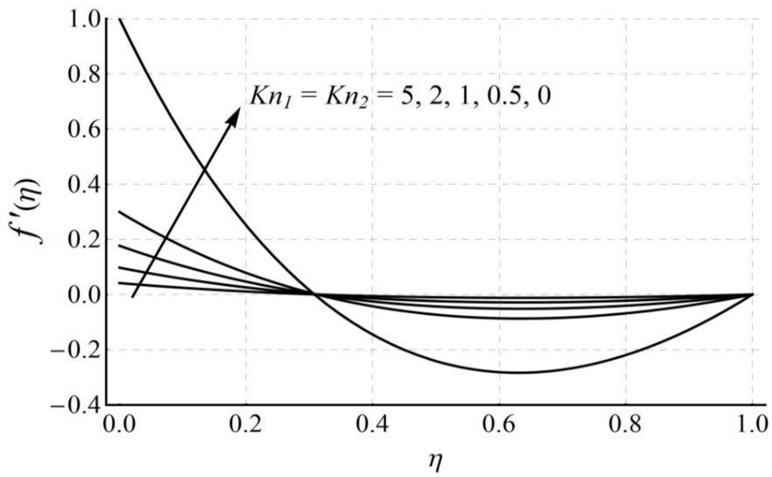


*Figure2.1 Impact of Kn on the velocity in the existence of AMT phenomenon for M=0.5, Re=2,w0=w1=0.5, β=0.5,*

**Fig.2.1**

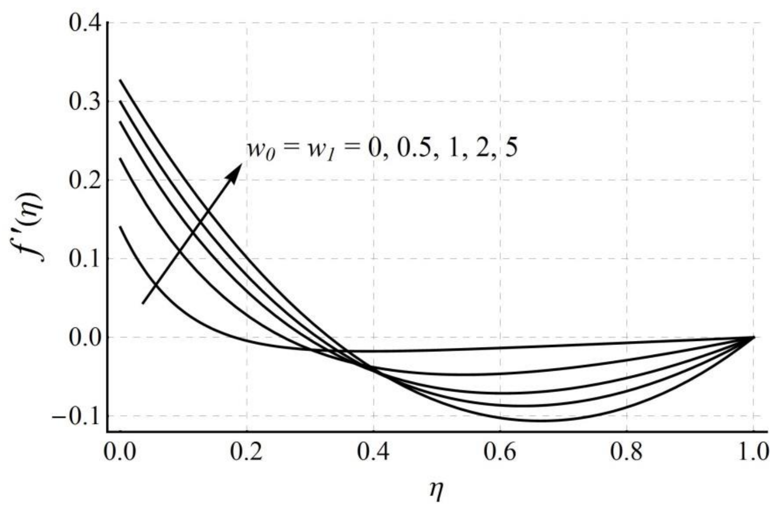


*Figure 2.2 Impact of AMT procedure on the velocity when M=0.5, β =0.5 , Kn1=Kn2=0.5 and Re=2,*

**Fig.2.2**

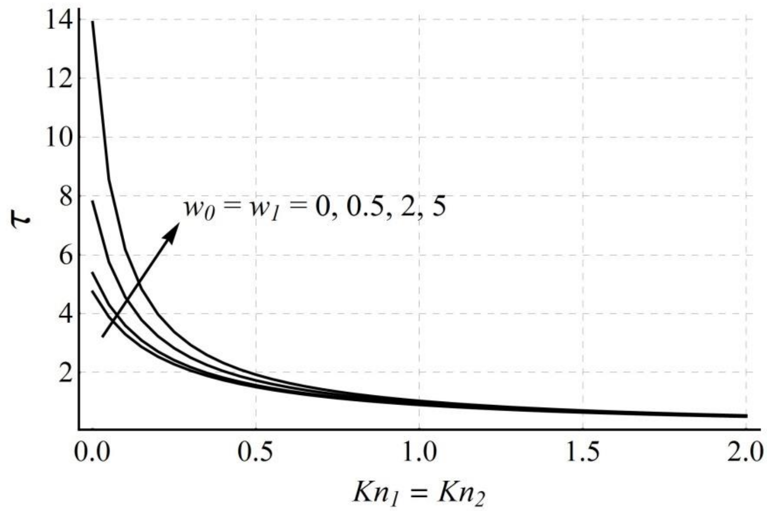


*Figure 2.3 Impact of AMT sensation on the SS against the slide factors while β =M=0.5 And Re=2*

**Fig.2.3**

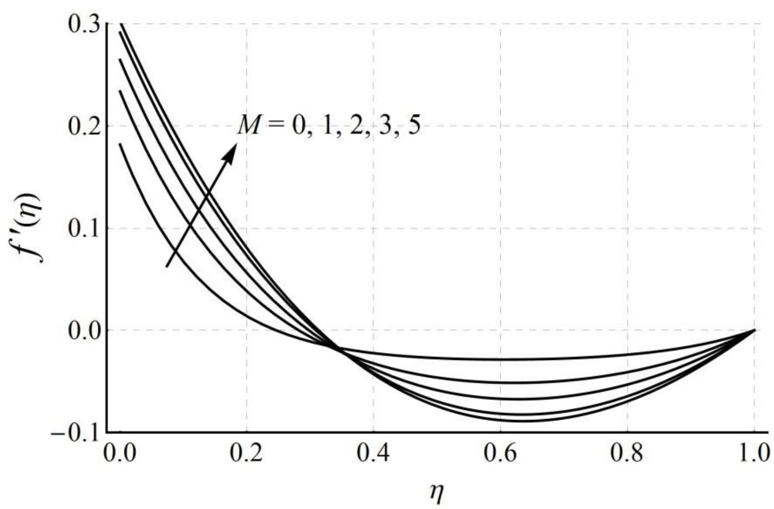


*Figure 2.4 Influence of the M on the velocity , while w0=w1=0.5 Re=2, β =0.5 and Kn1=Kn2=0.5,*

**Fig.2.4**

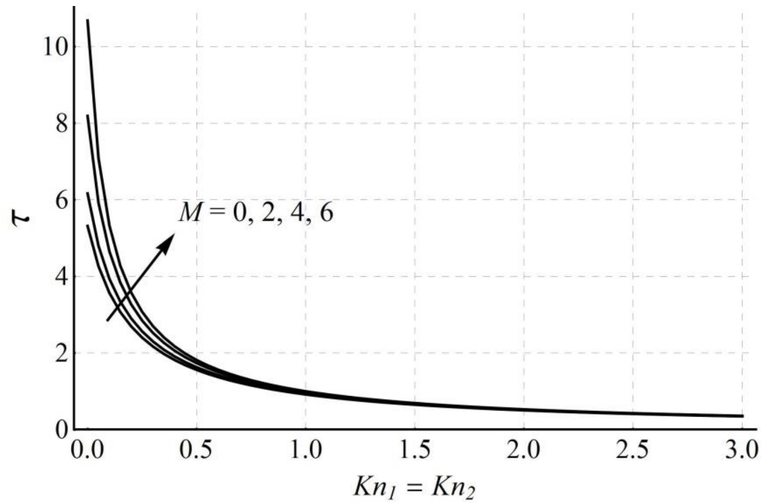


*Figure 2.5 Consequence of parameter M on the SS against the slide factors while β =w0=w1=0.5 and Re=2*

**Fig.2.5**

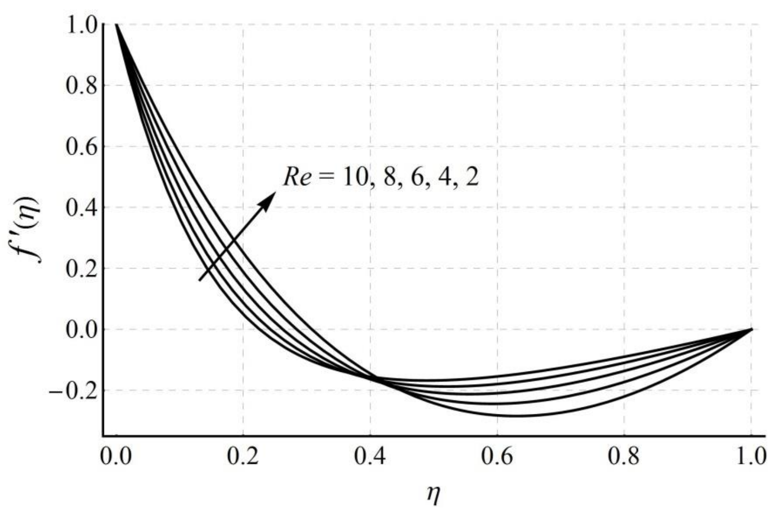


*Figure 2.6 Cause of Re on the velocity for w0=w1=0.5*

*and Kn1=Kn2=M=0.5*

**Fig.2.6**

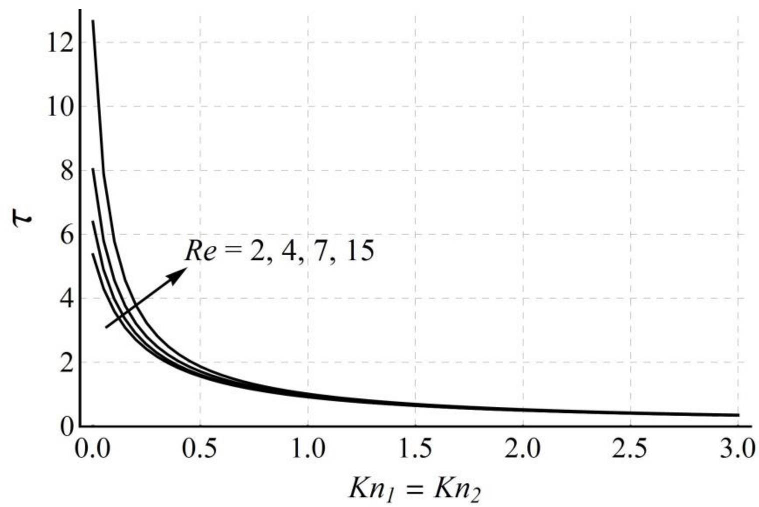


*Figure 2.7 Consequence of Re on the SS against the SC while w0=w1=0.5 and M= β =0.5*

**Fig.2.7**

## 2.5 Conclusion

From above analysis we have examined the impact of SC in MHD stream over an ST inside a duct within the sight of AMT phenomenon. The HAM is utilized to discover the investigative arrangements of exceedingly nonlinear diff. eqs. The HAM arrangement is likewise contrasted and the numerical arrangement by a SM and a decent assertion among the strategies has been established. It is presumed that the AMT phenomenon within the sight of SC is increasingly useful in diminishing and adjusting the viscid slog over the ST. Indeed, even within the sight of magnetism the viscid drag on the ST can be diminished by expanding the Kn. Thus the Kn is in opposite relation with velocity.

# Chapter#3

# Calculation of Temperature Distribution for Viscous Incompressible Flow with Constant Thermal Conductivity

## 3.1 Introduction

In our analysis we calculate the temperature conveyance in the presence of thermal conductivity with given bounds. The effects of a few parameters on temperature profile are investigated and pondered graphically. We present an absolutely analytic and exceptionally precise answer for the administering nonlinear eqs . We have utilized the HAM to explain the nonlinear eqs and looked at these outcomes by a numerical plan. HAM has incredible potential of dealing with exceptionally nonlinear eqs . Some commitments can be found in [2] and [28].

## 3.2 Mathematical Formulation of the Problem

We deliberate a VIF constrained by two unbounded corresponding even plates situated at $z=0$ and $z=h$ .It is presumed that lower plate is warmed at steady temperature $T_0$ and the upper plate is stable at temperature $T_h$ s.t $T_0 > T_h$. For this situation, the principal system contains the M. eqs as [30] and the E.eqs set by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\nu}{C_p}\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2+2\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right)\right] \tag{3.1}$$

With the appropriate bounds

$$\text{at } z=0 \quad T=T_0 \quad \text{and} \quad T=T_h \text{ at } z=h, \tag{3.2}$$

where $\nu$ is the viscosity and $k$ the thermic diffusivity of the liquid, $x, y$ and $z$ are space variables ,$C_p$ for the specific heat , Moreover $u, v$ and $w$ are velocity constituents, $h$ for the width of the channel, $T_h$ is the temperature at upper wall and $T_0$ the temperature at lower wall. We denote the similarity conversion as

$$\eta = \frac{z}{h}, \quad u = axF'(\eta), \quad v = ayG'(\eta), \quad w = -ah(F+G), \tag{3.3}$$

And $\theta(\eta) = \dfrac{T - T_h}{T_0 - T_h}$,

From (3.2) and (3.3) we take

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial^2 T}{\partial x^2} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial^2 T}{\partial y^2} = 0, \tag{3.4}$$

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial z} = \frac{(\mathrm{T}_0 - \mathrm{T}_h)\theta'}{h}, \quad \frac{\partial^2 T}{\partial z^2} = \frac{(\mathrm{T}_0 - \mathrm{T}_h)\theta''}{h^2} \tag{3.5}$$

$$\frac{\partial u}{\partial y} = \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = \frac{\partial v}{\partial x} = 0, \quad \frac{\partial u}{\partial z} = \frac{ax}{h}\mathrm{F''}(\eta), \quad \frac{\partial v}{\partial z} = \frac{ax}{h}\mathrm{G''}(\eta), \quad \frac{\partial w}{\partial z} = -a\left[\mathrm{F'}(\eta) + G'(\eta)\right]$$

And $\dfrac{\partial u}{\partial x} = aF''(\eta), \quad \dfrac{\partial v}{\partial y} = aG''(\eta)$ (3.6)

Substituting values from (3.4) to (3.6) into (3.1)

$$(\mathrm{F+G})\left[T_h - T_0\right]\theta' = -\frac{k}{a\rho C_p}\left(\frac{T_h - T_0}{h^2}\right)\theta'' + \frac{\upsilon}{aC_p}\left[\left\{\frac{a^2x^2}{h^2}\mathrm{F''}^2(\eta) + \frac{a^2y^2}{h^2}\mathrm{G''}^2(\eta)\right\} + 2\left\{2a^2\,\mathrm{F'}^2(\eta) + 2\mathrm{a}^2\mathrm{G'}^2(\eta) + 2\mathrm{a}^2\mathrm{F'}(\eta)\mathrm{G'}(\eta)\right\}\right] \tag{3.7}$$

$$\frac{k}{a\rho C_p}\theta'' + (\mathrm{F+G})\theta' - \frac{\upsilon}{ah^2}\left[\frac{a^2x^2}{C_P(T_h - T_0)}\mathrm{F''}^2(\eta) + \frac{a^2y^2}{C_P(T_h - T_0)}\mathrm{G''}^2(\eta)\right] + \frac{4av}{C_P(T_h - T_0)}\left[\mathrm{F'}^2(\eta) + G'^2(\eta) + F'(\eta)\mathrm{G'}(\eta)\right] = 0 \tag{3.8}$$

$$\theta''+(\mathrm{F}+G)\theta'\frac{a\rho C_p h^2}{k}-\frac{\rho C_p \upsilon}{k}\left[Ec_x\,\mathrm{F}''^2(\eta)+Ec_y\,\mathrm{G}''^2(\eta)\right]+4Ec\,\mathrm{Re}\left[\mathrm{F}'^2(\eta)+G'^2(\eta)+F'(\eta)\mathrm{G}'(\eta)\right]=0$$

(3.9)

$$\theta''-\frac{\mu C_p}{k}\left[(\mathrm{F}(\eta)+G(\eta))\theta'\frac{ah^2}{\upsilon}+\left(Ec_x\,\mathrm{F}''^2(\eta)+Ec_y\,\mathrm{G}''^2(\eta)\right)+4Ec\,\mathrm{Re}\left[\mathrm{F}'^2(\eta)+G'^2(\eta)+F'(\eta)\mathrm{G}'(\eta)\right]\right]=0$$

(3.10)

After that we have the following

$$\theta''=\mathrm{Pr}\left[\mathrm{Re}\left(F+G\right)T'+4Ec\,\mathrm{Re}\left(\mathrm{F}'^2+G'^2+F'\mathrm{G}'\right)+Ec_x\,\mathrm{F}''^2+Ec_y\,\mathrm{G}''^2\right] \qquad (3.11)$$

With transformed boundary condition

$$\theta(0)=1, \qquad \theta(1)=0, \qquad (3.12)$$

here $\mathrm{Pr}=\mu C_p/k$ is the Prandtl number, $'$ symbolizes the diversity with respect to $\eta$, $\mathrm{Re}=ah^2/\nu$ is the Reynolds number, $E_c=\nu a/C_p\left(T_w-T_0\right)$ for Eckert number, $Ec_x=a^2x^2/C_p\left(T_w-T_0\right)$ and $Ec_y=a^2y^2/C_p\left(T_w-T_0\right)$ are the local Eckert numbers, respectively. Now we focus on the HTP. Thus the eqs and the given bounds for the elaborated velocity components have not been given here so as to make the composition reduced.

## 3.3 Analytic HAM Solution

So as to get the temperature profile we will need to understand the fundamental scheme. We utilize HAM to resolve this scheme of nonlinear eqs. Furthermore, the essential estimate approximations fulfilling the eq 3.12 as following

$$\theta_0(\eta)=1-\eta, \qquad (3.13)$$

In perspective on boundary conditions 3.12, we take the linear operator as

$$L_T=\frac{d^2}{d\eta^2} \qquad (3.14)$$

Since HAM is notable and is broadly utilized by the network of fluid dynamics , temperature and mass exchange.

### 3.3.1 Mth- Order Distortion

Mth order distortion eq $(m \geq 1)$ is given by

$$L_T\left[T_m(\eta)-\chi_m T_{m-1}(\eta)\right]=hH_m(\eta) \tag{3.15}$$

Insert to the bounds given below

$$T_m(0)=1, \quad T_m(1)=0, \tag{3.16}$$

Where

$$H_m(\eta)=T''_{m-1}(\eta)+$$
$$\Pr\mathrm{Re}\sum_{k=0}^{m-1}\begin{pmatrix}F_{m-1-k}(\eta)\\+G_{m-1-k}(\eta)\end{pmatrix}T'_k+4Ec\Pr\mathrm{Re}\sum_{k=0}^{m-1}\begin{bmatrix}\mathrm{F}'_{m-1-k}\mathrm{F}'_k+G'_{m-1-k}\mathrm{G}'_k\\+F'_{m-1-k}\mathrm{G}'_k\end{bmatrix}+\Pr Ec_x\sum_{k=0}^{m-1}\begin{bmatrix}\mathrm{F}''_{m-1-k}(\eta)\\\mathrm{F}''_k(\eta)\end{bmatrix}+\Pr Ec_y\sum_{k=0}^{m-1}\left[\begin{pmatrix}\mathrm{G}''_{m-1-k}(\eta)\\\mathrm{G}''_k(\eta)\end{pmatrix}\right], \tag{3.17}$$

And

$$\chi_k=\begin{cases}0, k\leq 1,\\1, k>1,\end{cases} \tag{3.18}$$

Where $T_m(\eta)$ is the result of the mth order $(m \geq 1)$ distortion eq.

### 3.3.2 Convergence of HAM Solution

As referenced by, the cvg of HAM series intensely be influenced by the value of the cvg controlling parameters, namely, $h$.In HAM results the higher order of estimations are the amendments to these results, and for a congregating sequences the modifications must lie in a collective order of calculations. We also deliberated such type of modifications (see table 3.1) for the existing problem. From table 3.1 obviously the revisions to the HAM arrangement become immaterial at high number of calculations. This verifies the cvg of HAM result.

**Table 3.1** Convergence of HAM when h=-0.8, pr=0.71, $Ec = Ec_x = Ec_y = 0.2$ are reserved fixed.

| Re | $\beta$ | $w_0 = w_1$ | $f''(0)$ | $T'(0)$ |
|---|---|---|---|---|
| 1 | 0.5 | 0.5 | -0.811861 | -0.846939 |
| 2 | 0.5 | 0.5 | -0.823103 | -0.696192 |
| 3 | 0.5 | 0.5 | -0.833653 | -0.567555 |
| 2 | 1 | 0.5 | -0.823296 | -0.705960 |
| 2 | 2 | 0.5 | -0.823676 | -0.750082 |
| 2 | 0.5 | -0.5 | -0.785078 | -1.40192 |
| 2 | 0.5 | 0.0 | -0.804118 | -1.00764 |

## 3.4 Numerical outcomes and discussions

So as to research the impact of temperature distribution with constant thermal conductivity, graphs are plotted in figures 3.1-3.11.We have plotted the temperature profile for various factors and observed the behavior by varying the values of those factors. Fig 3.1 shows the temperature distribution at different values of $w_1$ by keeping Ec zero i.e. ($Ec = Ec_x = Ec_y = 0$), as the values of $w_1$ rises, the temperature distribution also increases. Fig 3.2-3.4 are intrigued to observe the impacts of the congregating value of temperature by varying Ec. It is seen that as the Ec increases, the temperature profile decreases. Fig 3.5 illustrates the influence of velocity gradients $\beta = b/a$ on temperature. It is noticed that when we vary the values of $\beta$ from low to high, the temperature also rises. This indicates the direct relation with temperature. In fig 3.6 and 3.7, it is perceived that the consequence of Kn1 and Kn2 on $T(\eta)$ .It is detected that the temperature of the fluid increases as the Kn increases. Fig 3.8 shows that in the case of SC the influence of growing hydro- magnetic force on the flow. The fig demonstrates the variation trend in the temperature profile.. As M increases, the temperature profile increases from top to bottom . As M increase the SS also increase which is because of the increase in the resistive force to the flow. Fig 3.9 describes the behavior of temperature by varying Pr. If Pr increases the temperature decreases. For small Prandtl number, Pr << 1, means the thermal diffusivity dominates. For large, Pr >> 1, depict that the magnetic diffusivity leads the behavior. However the graphic values are converging .In fig 3.11 it is observed that increase in injection velocity at the upper portion and suction velocity at the lower portion results an increase in the viscous hindrance on the ST. It is perceived that this viscous hindrance can be decreased by raising the value of Kn. The resulting temperature values are congregating by showing increasing pattern with an increase in the value of w0.

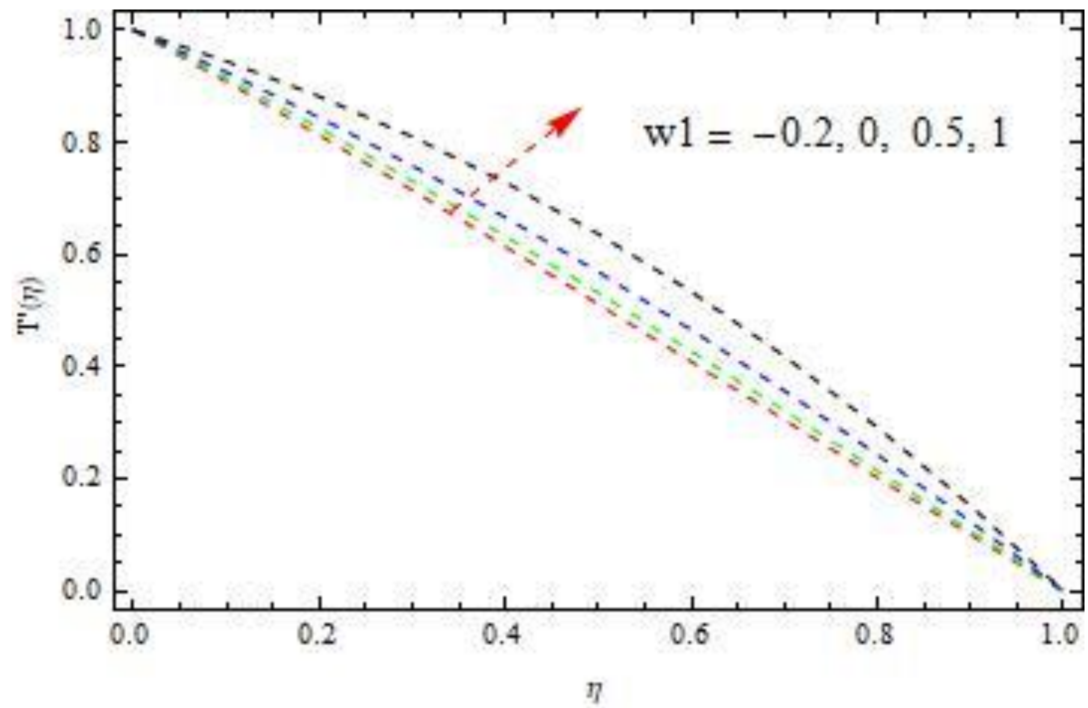


Figure 3.1 temperature at different w1 when w0=β=Kn1=Kn2=1/2, Pr=71/100, Re=2

**Fig. 3.1**

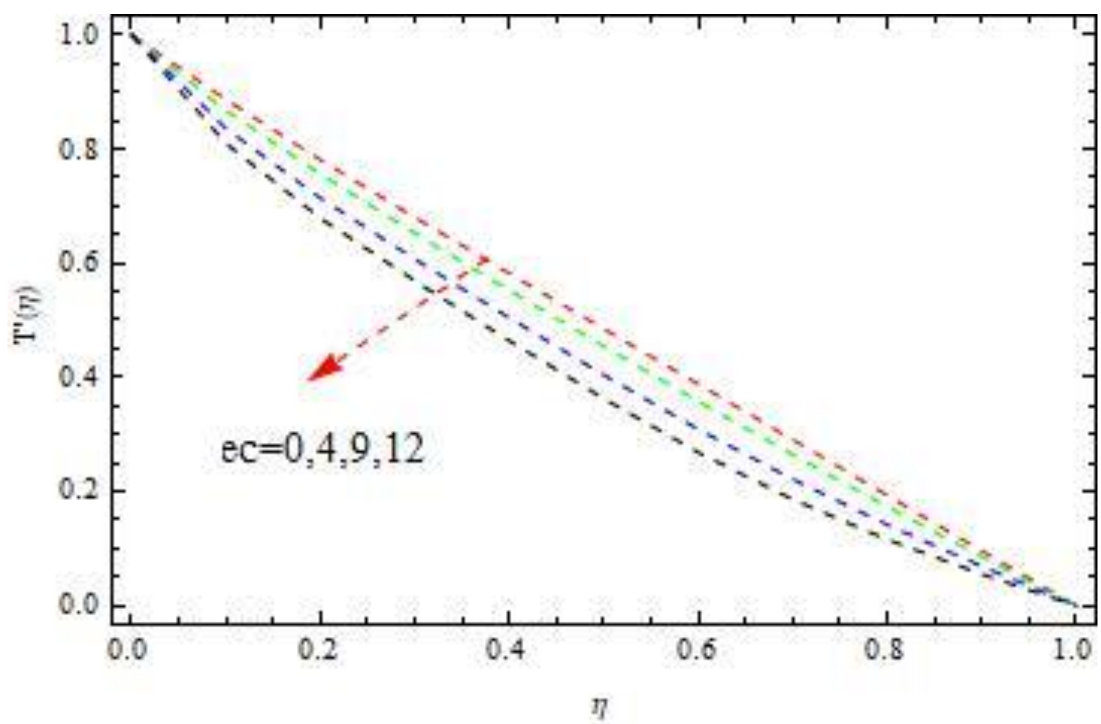


*Figure 3.2 Impact of Eckert number on temperature at w0=w1=0 ,β=Kn1=Kn2=1/2, Re=2,Pr=71/100,* ***Fig .3.2***

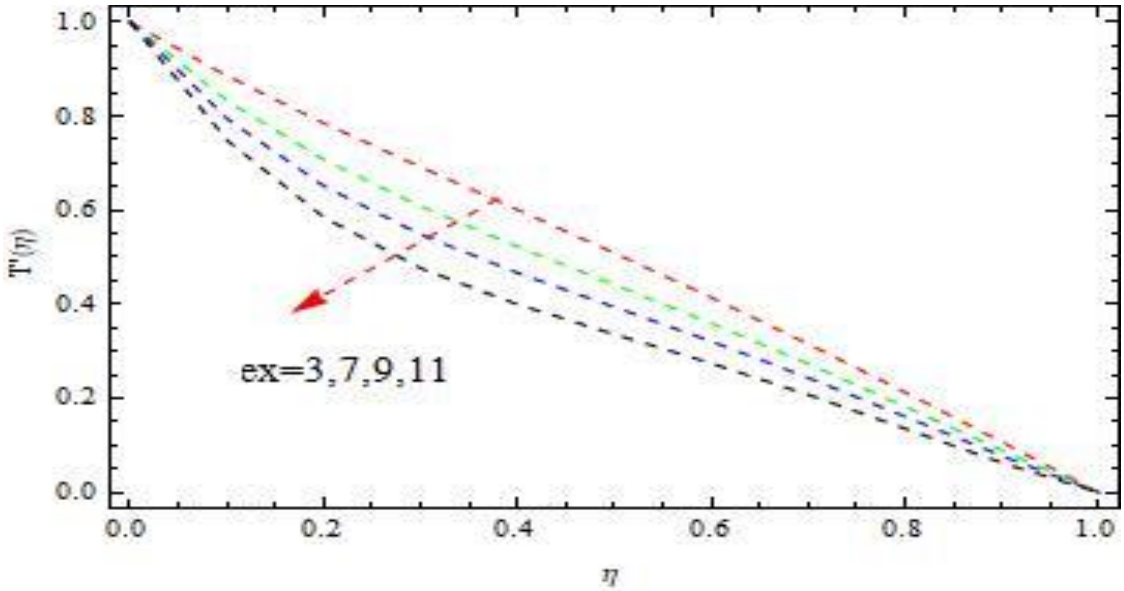


*Figure 3.3 Impact of local Ec on temperature profile Re=2,Pr=71/100,β=w0=w1=1/2*

**Fig .3.3**

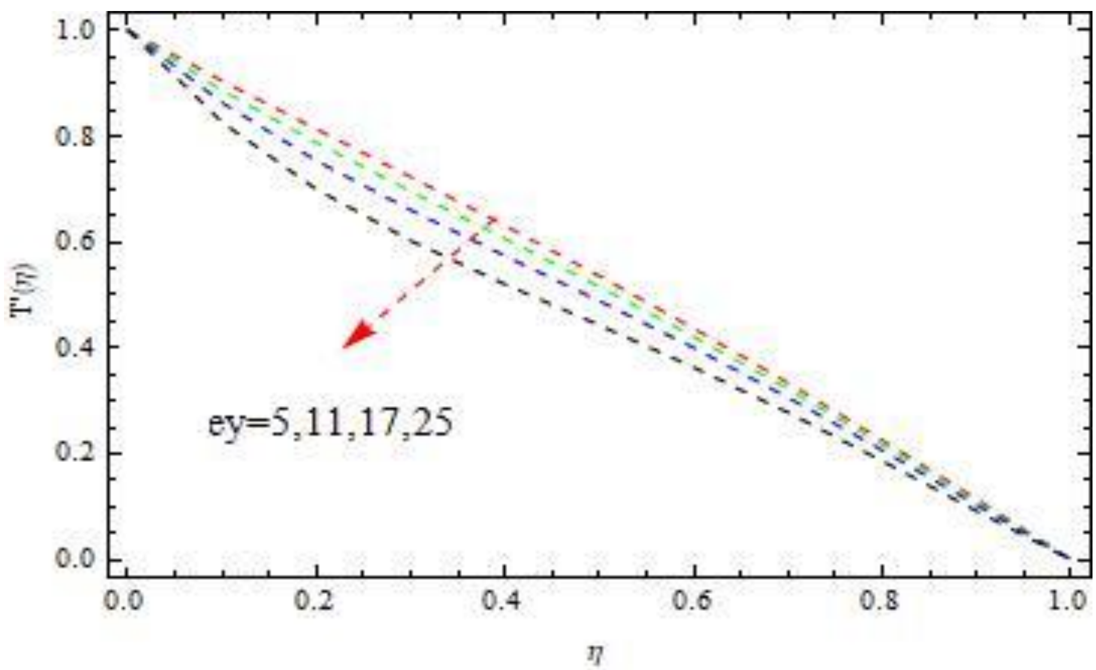


*Figure 3.4 Effect of local Eckert number on Temperature, Re=2,Pr=71/100,β=w0=w1=1/2*

**Fig .3.4**

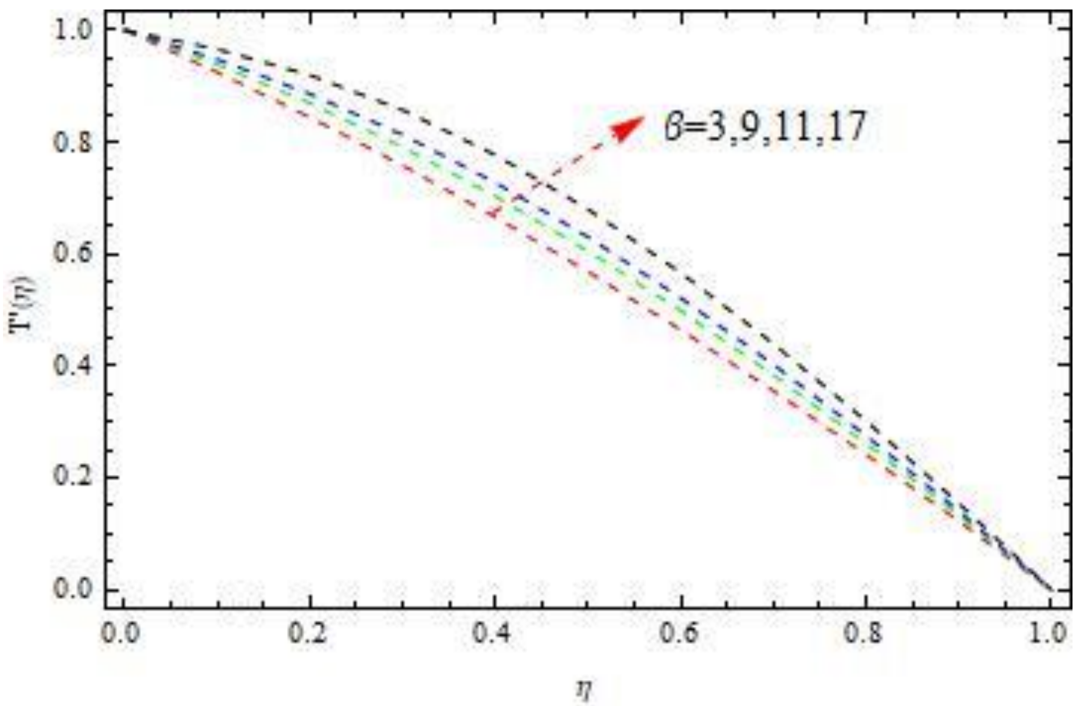


*Figure 3.5 Influence of β on temperature*
*Re=2.Pr=71/100,w0=w1=1/2*

**Fig .3.5**

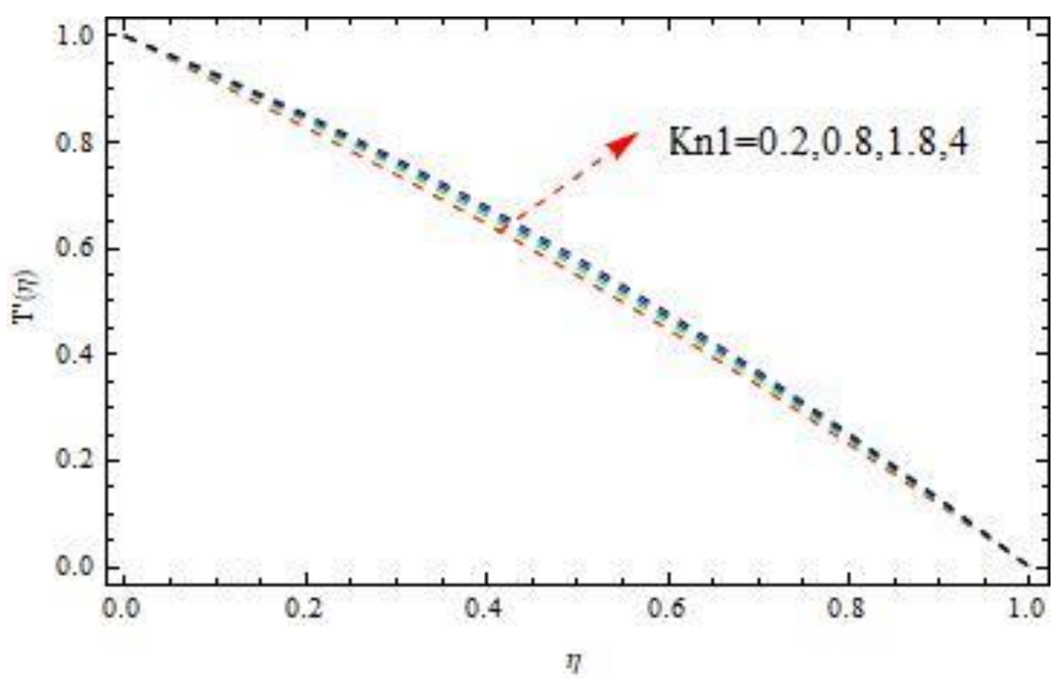


*Figure 3.6 Consequence of Kn on temperature profile*
*Re=2,Pr=71/100,ex=ey=ec=1/5,w0=w1=1/2*

**Fig.3.6**

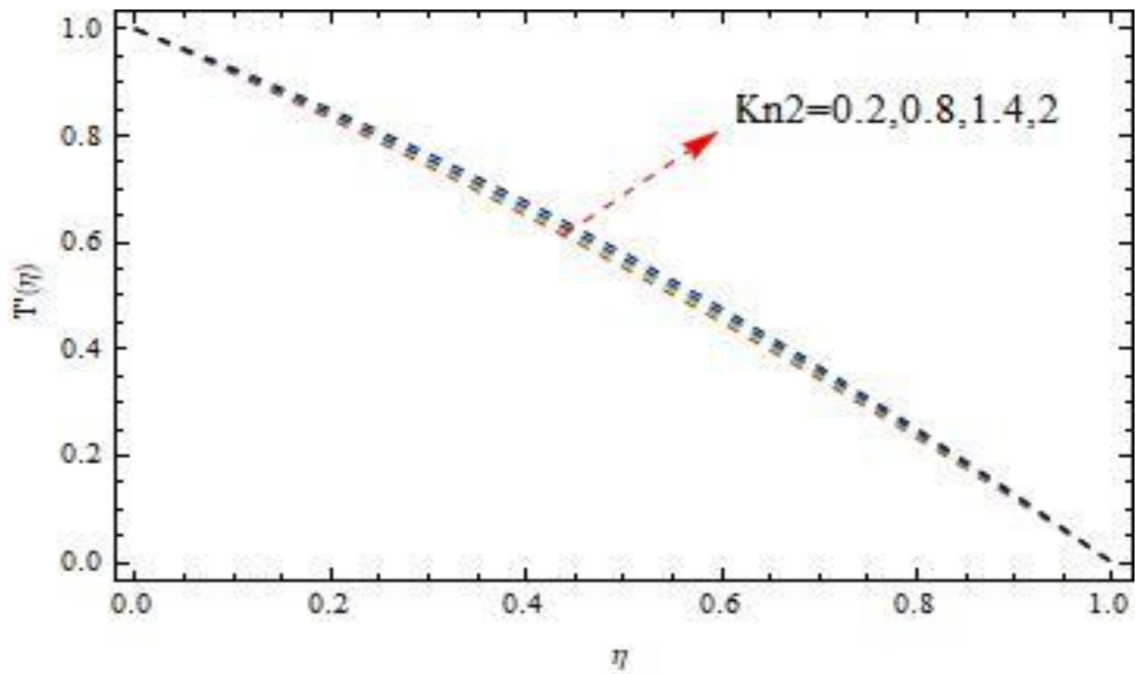


*Figure 3.7 Consequence of Kn on temperature ,w0=w1=β=1/2,Re=2,Pr=71/100*

**Fig.3.7**

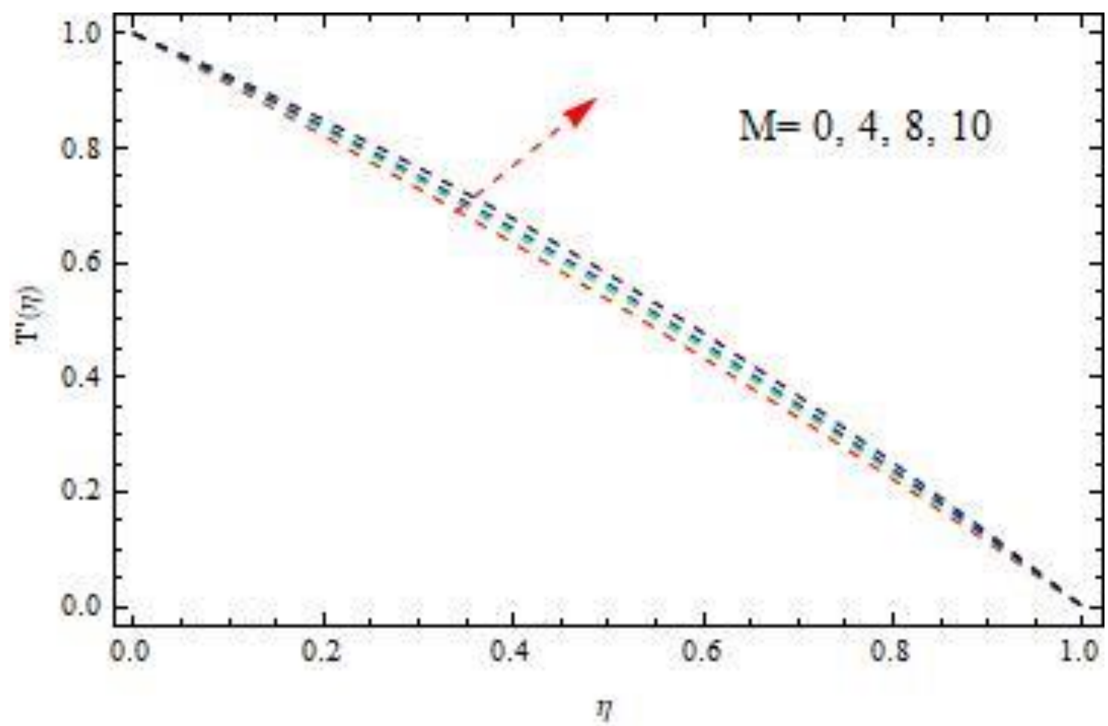


*Figure 3.8 Influence of M on temperature ,Re=2,Pr=71/100,β=w0=w1=1/2, ex=ey=ec=1/5*

***Fig.3.8***

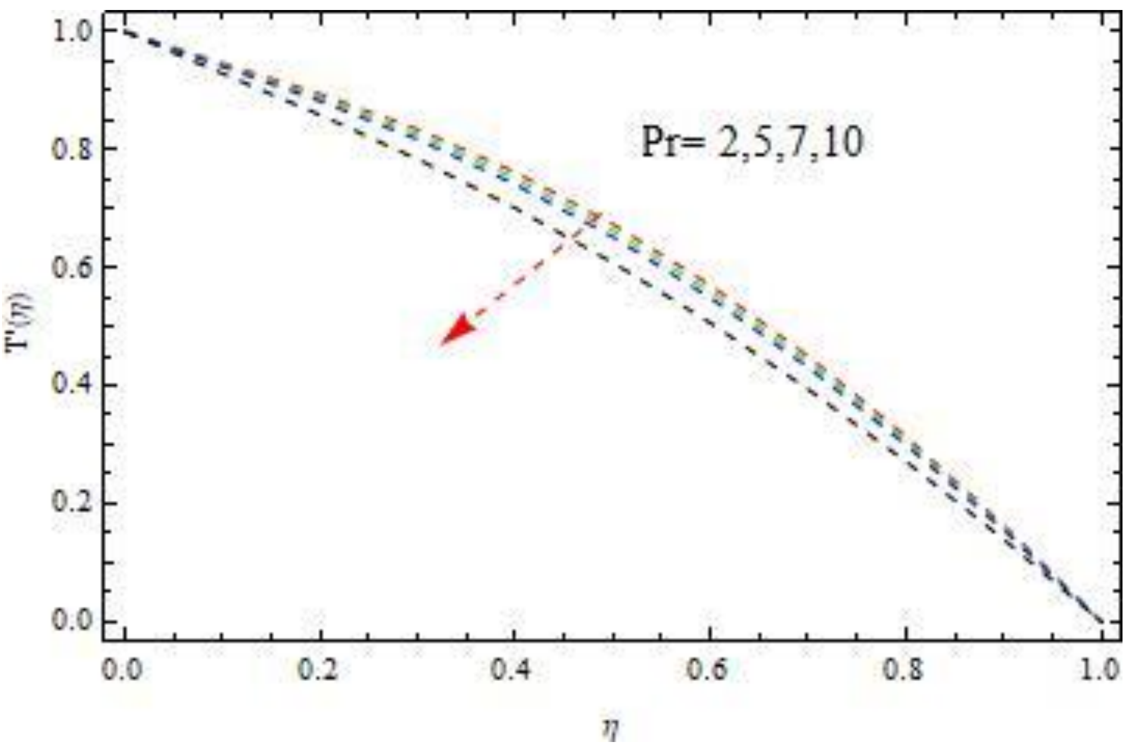


Figure 3.9 Influence of Pr on temperature ,Re=2,β=w0=w1=Kn1=Kn2=1/2,

**Fig.3.9**

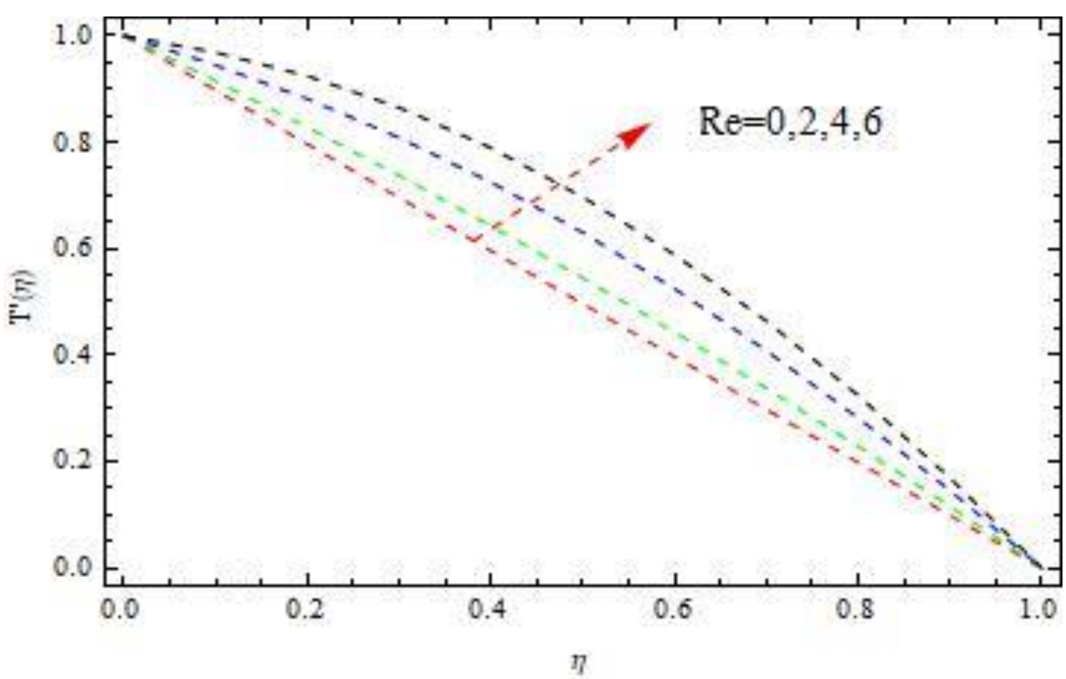


Figure 3.10 Temperature at different values of Re ,Pr=71/100,w0=w1=β=1/2

**Fig.3.10**

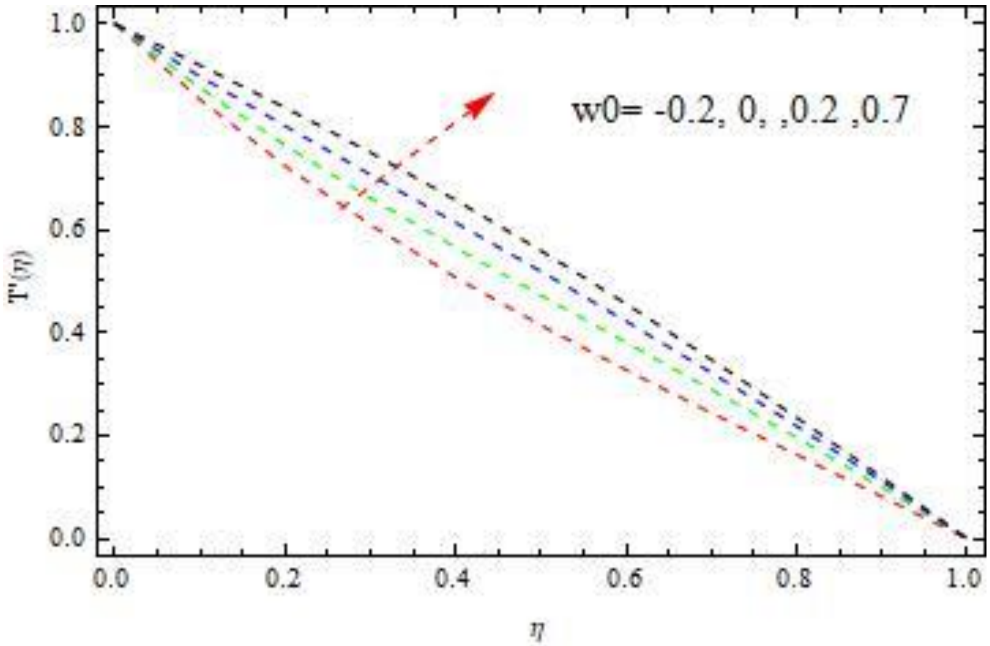


*Figure3. 11 Temperature at different w0, Re=2,Pr=71/100 Kn1=Kn2= β =ec=ex= ey=1/2*

**Fig.3.11**

## 3.5 Conclusion

We have inspected temperature distribution with constant thermal conductivity inside a channel .Behavior of temperature profile is noticed by changing different constraints .The HAM is used to find the analytical solution of exceedingly nonlinear diff. eq. Because of the elasticity of HAM in free choice for the variety of constraint values, it is detected that this method was appropriate and high precision method for resolving non-linear diff.eqs. . Reasonable clarifications are taken by utilizing graphs demonstrating the behavior of parameters. Also it is deduced that Knudsen number is in direct relation with temperature.

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